MATH317	**MID-TERM EXAM**	**Name:** ________________
Spring 1999		

Note: [1] There should be two parts.

[2] Read the questions carefully before answering.

[3] Show **all** your work to receive credit.

PART A (15 points)

State **true or false** or **fill in the blanks** with appropriate answers, for each of the following statement (assume all matrix operations are valid).

(1) When both A and B are nonsingular $(AB)^{-1} = B^{-1} A^{-1}$.

(2) A is nonsingular if and only if det(A) is nonzero.

(3) Rank (AB) is always equal to rank of A.

(4) det(A + B) = det(A) + det(B).

(5) If A and B are square matrices, then det(AB) = det(A)det(B).

(6) When A inverse exists, $\det(A^{-1}) = 1/\det(A)$.

(7) A square matrix A is said to be skew-symmetric iff

(8) The system A**x** = **b** will have solution iff rank (A) = rank[A **b**].

(9) It is possible for the system A**x** = **0** to be inconsistent.

(10) W = $\{(x,0)^T$: x is a real number$\}$ is a subspace of R^2 with standard operations.

(11) V = {set of all 3 $\times$ 3 nonsingular matrices} is a subspace of the set of all 3 x 3 matrices with standard operations.

(12) For the matrix $A = \begin{bmatrix} i & 2 & -5 \\ 0 & 2 & 4+i \\ 1 & 5 & -8 \end{bmatrix}$ the trace is ____ and the determinant is

(13) Suppose that **x** is a nonzero vector. Then A**x** = **0** iff A is singular.

(14) A Adj(A) = det(A) I is always true when and only when A is nonsingular.

(15) It is possible for a p x q matrix A to have neither a left- nor a right-inverse.

PART B (85 points)

[1] Given $z_1 = 1 + i$, $z_2 = i - 4$, and $z_3 = 4 + 3i$, find

(a) $z_1 + z_2 - 4z_3$

(b) $|z_1 + z_2 - 4z_3|$

(c) the complex conjugate of $z_1 + z_2 - 4z_3$.

[2] Show that for all complex numbers z and w $\overline{zw} = \overline{z}\ \overline{w}$

[3] Suppose that A and B are p x p matrices such that AB is nonsingular. Show that A and B both are nonsingular.

[4] Show that B is row-equivalent to A. Show that there exists a nonsingular matrix F such that FA = B.

[5] Solve the system of linear equations AX = B, where A, X, and B are given by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix},$$

[6] Show that every Hermitian matrix is the sum of a real symmetric matrix and i times a real skew-symmetric matrix.

[7] Suppose that A is a nonsingular matrix of order 3, reduced to an identity matrix by performing the following elementary row operations (in the same order): (i) $E_1(-1)$; (ii) $E_{21}(1)$; (iii) $E_2(0.5)$; Find the matrix A^{-1}, by first writing A as a product of elementary matrices.

[8] Suppose that B is p x q and A is a p x p nonsingular matrix. Show that the rank of AB equals the rank of B.

[9] Given the matrix

$$A = \begin{bmatrix} 1 & 2 & -6 \\ -3 & 4 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

find the LU decomposition of A

[10] If A is a square matrix of order p, show that

(a) A Adj(A) = det(A) I

(b) $\det(\mathrm{Adj}(A)) = (\det(A))^{p-1}$.